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ABSTRACT

This study examines the responses of security and portfolio returns to up- and down-markets. The variance of portfolio returns is divided into up- and down-market segments based on the hypothesis that investors expect a premium for downside risk but will pay a premium for upside variation. The results suggest securities and portfolios do respond differently in up- and down-markets and that investors are cognizant of differences in upside and downside risk.

## AN ANALYSIS OF NONSYMMETRIC SYSTEMATIC RISK

### I. INTRODUCTION

In recent years the beta coefficient in the capital asset pricing model developed by Sharpe, Lintner, and Mossin [6, 3, 5] has won wide acceptance in the academic community as a relevant measure of risk. It is asserted that the return of a portfolio or asset should not be a function of its total risk, but that return should only contain a risk premium for that portion of total risk which cannot be eliminated by diversification. The beta coefficient is a measure of this nondiversifiable or systematic risk when the asset's return is related to the return of some market index. Because assets with high betas have a high degree of systematic risk which cannot be eliminated by diversification, these assets must be expected to provide higher returns in order to compensate for the higher risk. Conversely, low beta assets exhibit lower degrees of systematic risk and, hence, are expected to provide lower returns to the investor. The capital asset pricing model (CAPM) can be utilized in the trading of assets when expectations change regarding the movement of the market. Returns on low beta assets are expected to decline less than the market when the market falls and to increase less when the market rises. High beta assets are expected to provide returns greater than the market in rising markets and returns less than the market when the market is falling. Therefore, investors prefer to hold low beta (low risk) assets when the market is expected to fall and to hold high beta (high risk) assets when the market is expected to rise.

The problem of trading assets may be alleviated by selecting assets with nonsymmetric betas. The CAPM assumes the returns of an asset relative

to the market is the same regardless of the direction of the market movement. This assumption results in each asset having a single symmetric beta, and implies the systematic risk of an asset is the same in up-markets as in down-markets.

If an asset performs differently in up- and down-markets, the single beta is not an appropriate measure of the risk of the asset. An appropriate measure must consider the possibility of an asset having a nonsymmetric beta; that is, the possibility that an asset responds differently in up- and down-markets. Therefore, the asset's response in both up- and down-markets must be examined. This can be done by calculating two betas; one for up-markets and one for down-markets. The up-market beta, $b_i^+$, is a measure of the systematic risk when the market is rising while the down-market beta, $b_i^-$, measures the sytematic risk in down-markets. The purpose of this study is to investigate the hypothesis of nonsymmetric betas and to examine its implication with respect to capital market theory.

## II. THE TWO BETA MODEL

Two assumptions are utilized in the development of the study. First, it is assumed that each security may respond differently in up- and down-markets. If this is the case, beta coefficients may be determined for both types of markets and examined for statistically significant differences. Three alternative measures were used to determine what constituted an "up" or "down" market. An "up" market was defined as those months when the rate of return on the market portfolio exceeded (1) the average market return, (2) the risk free rate, or (3) zero. Otherwise, the market was defined as a "down" market.

Under this assumption, consider the following security characteristic line:

$$R_i = \alpha_i + \beta_i^+ R_m^+ + \beta_i^- R_m^- + e_i \qquad (1)$$

where $R_i$ = the random rates of return on security i,

$R_m$ = the random rates of return on a market index,

$$R_m^+ = \begin{cases} R_m & \text{if } R_m > d \\ 0 & \text{if } R_m \leq d \end{cases}$$

$$R_m^- = \begin{cases} R_m & \text{if } R_m < d \\ 0 & \text{if } R_m \geq d \end{cases}$$

d = the level of return used to distinguish an up-market from a down-market, i.e., the average market return, the average risk free rate ( 3 mo. T-Bills), or zero.

$e_i$ = random errors with the following properties; $E(e_i) = 0$ and $E(e_i, e_j) = 0$ for $i \neq j$.

$\alpha_i$ = a regression constant.

The return on a portfolio composed of a number of securities (i = 1, 2, . . ., n) with corresponding proportions of $w_i$ where $\sum_i w_i = 1$, becomes

$$R_p = \sum_i w_i \alpha_i + (\sum_i w_i \beta_i^+) R_m^+ + (\sum_i w_i \beta_i^-) R_m^- + \sum_i w_i e_i . \qquad (2)$$

The expected return and variance of the portfolio are

$$E(R_p) = \alpha_p + \beta_p^+ E(R_m^+) + \beta_p^- E(R_m^-) \qquad (3)$$

and $$\sigma^2(R_p) = (\beta_p^+)^2 \sigma^2(R_m^+) + (\beta_p^-)^2 \sigma^2(R_m^-) + \sigma^2(e_p), \qquad (4)$$

where $\alpha_p = \sum_i w_i \alpha_i$

$E(\sum_i w_i e_i) = 0$

$\beta_p^+ = \sum_i w_i \beta_i^+$

$\beta_p^- = \sum_i w_i \beta_i^-$

$\sigma^2(e_p) = \sigma^2(\sum_i w_i e_i)$

Remember from equation (1) that the $\beta_p^+$ is calculated from the data when $R_m > d$ while $\beta_p^-$ is calculated from the returns when $R_m < d$.

Since as the number of securities in a portfolio increases the unsystematic portion of the variance, $\sigma^2(e_p)$, is being diversified away, equation (4) may be rewritten as;

$$\sigma^2(R_p) \cong \underbrace{(\beta_p^+)^2\sigma^2(R_m^+)}_{A} + \underbrace{(\beta_p^-)^2\sigma^2(R_m^-)}_{B}. \qquad (5)$$

The variance of the portfolio has now been decomposed into two parts:

A is that part of the total variance systematically related to the up-market, and

B is that part of the total variance systematically related to the down-market.

The second assumption invoked in the study is that investors prefer a larger upside variation of returns to a smaller one, and prefer a smaller downside variation of returns to a larger one. This implies that investors' preference is positively related to the size of A in equation (5), but negatively related to the size of B. By treating separately the upside and downside variations from the total variation, we depart from the conventional assumption that the larger the total variance of the portfolio the less the portfolio is preferred. Instead, we hypothesize that investors require a risk premium on the downside portion of the variation and a negative risk premium on the upside portion of the variation. This may be expressed as;

$$E(R_p) = R_f + C_1[\beta_p^+\sigma(R_m^+)] + C_2[\beta_p^-\sigma(R_m^-)], \qquad (6)$$

where $R_f$ = the risk free rate

$C_1$ = a negative coefficient

$C_2$ = a positive coefficient

$$\text{or} \quad E(R_p) = R_f + \lambda_1\beta_p^+ + \lambda_2\beta_p^- \tag{7}$$

where $\lambda_1 = C_1\sigma(R_m^+)$

and $\lambda_2 = C_2\sigma(R_m^-)$.

It is not difficult to see that the single beta capital asset pricing model is a special case of the hypothesized model in equation (7). For the market portfolio (or well diversified portfolios) which has $\beta_m^+ = \beta_m^- =$ 1.0, equation (7) becomes

$$E(R_m) = R_f + \lambda_1 + \lambda_2$$

$$\text{or} \quad \lambda_1 + \lambda_2 = E(R_m) - R_f. \tag{8}$$

Thus, any portfolio with symmetric beta values ($\beta_p^+ = \beta_p^- = \beta_p$) has the following relationship from equations (7) and (8);

$$E(R_p) = R_f + (\lambda_1 + \lambda_2)\beta_p$$

$$= R_f + [E(R_m) - R_f]\beta_p \tag{9}$$

This is the familiar relationship of the single beta capital asset pricing model.

The next section presents the results of the tests for (1) the assumption of nonsymmetric betas and (2) the assumption of positive and negative risk premiums for the downside and upside portions of total risk, respectively.

## III. TESTS OF THE TWO BETA-MODEL

### Data

The sample was taken from monthly closing stock prices on the Compustat Price, Dividend and Earnings tape. The period analyzed was the 179 month period from February, 1962 through December, 1976. A total of 322 securities had data available for the entire period. The Standard and Poor's 500 Index was used as the market portfolio.

Monthly returns were calculated as

$$R_t = \frac{P_{t+1} + D_t - P_t}{P_t} \times 100$$

where $R_t$ = the return for month t,

$P_t$ = the beginning monthly price,

$P_{t+1}$ = the ending monthly price, and

$D_t$ = the cash dividends paid during the month.

Tests of the Nonsymmetry of Betas

The monthly rates of return on the market index were divided into up- or down-markets using three alternative cutoff levels; (1) the average monthly market return (0.619%), (2) the average monthly yield on three-month Treasury Bills (0.407%), and (3) zero. Using each of the three cutoff levels in turn, coefficients of the two-beta model of equation (1) were estimated for each of the 322 securities. Because the different cutoff levels resulted in virtually identical results, only the results associated with the average monthly market return are reported.

Table 1 here

The distribution of the the regression estimates, $b_i^+$ and $b_i^-$, of $\beta_i^+$ and $\beta_i^-$ are shown in Table 1. The main diagonal cells represent those securities with $b_i^+$ and $b_i^-$ in the same relative class. Those securities which respond more in up-markets than down-markets, $b_i^+ > b_i^-$, are located below the diagonal while those with opposite characteristics, $b_i^+ < b_i^-$, are above the diagonal. Figure 1 shows the distribution of the difference between $b_i^+$ and $b_i^-$ for the 322 security sample. The Kolmogorov-Smirnov test for normality indicates the distribution is normal at the 0.01 level.

Figure 1 here

TABLE 1

THE DISTRIBUTION OF UP- AND DOWN-MARKET BETAS FOR 322 SECURITIES

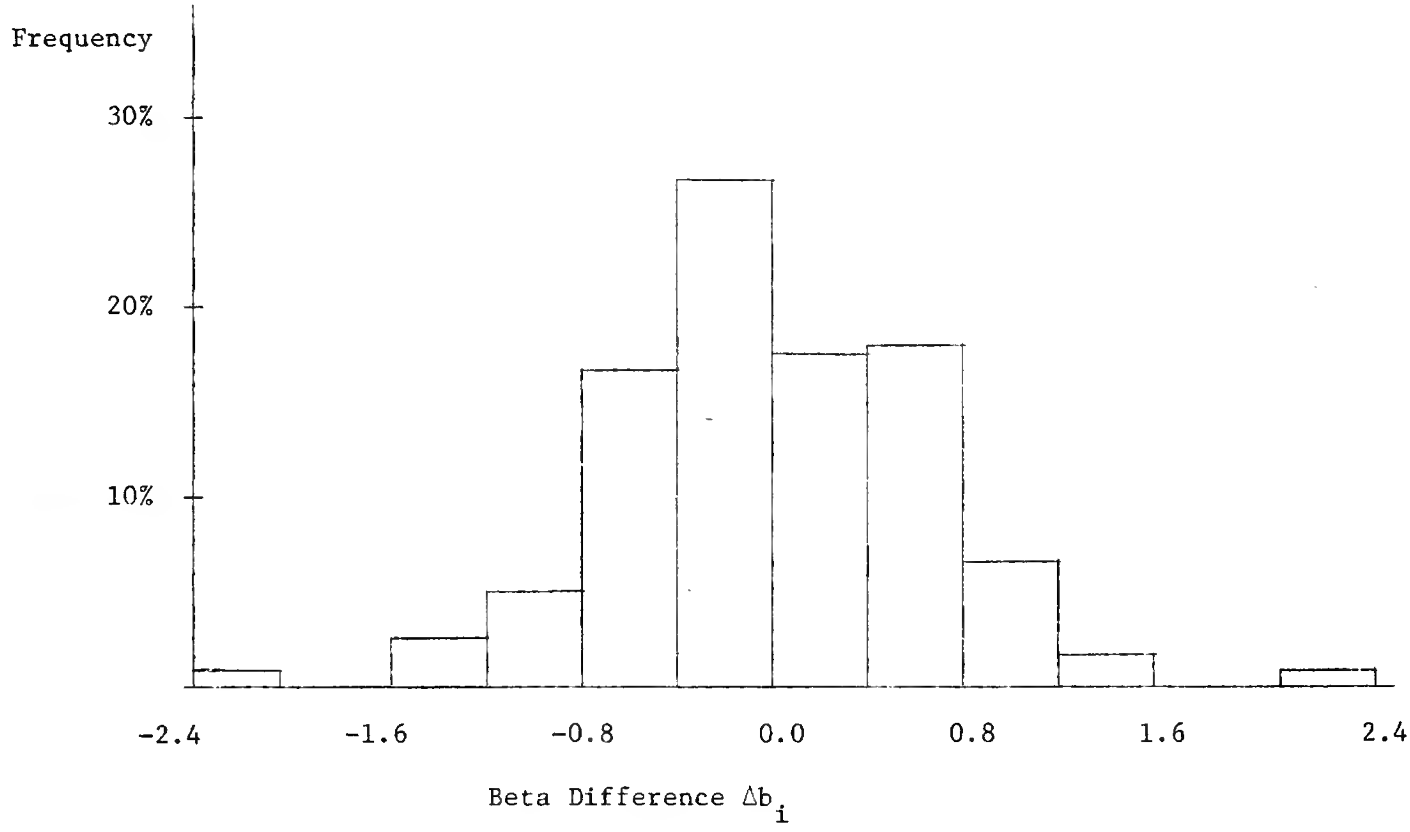
| Up-Market Beta $b_i^+$ | Down-Market Beta $b_i^-$: 0.0 | 0.2 | 0.6 | 1.0 | 1.4 | 1.8 | 2.2 | 2.6 | 3.0 | | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | | | | | | | 1 | | | | 1 |
| 0.2 | | | | | | | | | | | 0 |
| 0.6 | | | 3 | 4 | 7 | 6 | 1 | | 1 | | 23 |
| 1.0 | 2 | 1 | 4 | 18 | 35 | 15 | 5 | 2 | 1 | | 83 |
| 1.4 | | 1 | 8 | 38 | 40 | 25 | 10 | 4 | | | 126 |
| 1.8 | | 1 | 7 | 20 | 14 | 11 | 8 | 3 | 1 | | 65 |
| 2.2 | | | 1 | 5 | 4 | 4 | 4 | 1 | | | 19 |
| 2.6 | | | | 1 | 1 | 1 | | 1 | | | 4 |
| 3.0 | | | | | | 1 | | | | | 1 |
| Total | 2 | 3 | 23 | 86 | 101 | 63 | 29 | 12 | 3 | | 322 |

| | $b_i^+$ | $b_i^-$ |
|---|---|---|
| Beta Means | 1.18 | 1.22 |
| Beta Standard Deviations | 0.42 | 0.52 |

FIGURE 1

THE DISTRIBUTION OF THE DIFFERENCES BETWEEN UP- AND DOWN-MARKET BETAS
FOR 322 SECURITIES

$(\Delta b_i = b_i^+ - b_i^-)$




| | |
|---|---|
| Mean $(\Delta b_i)$ | -0.03 |
| Standard Deviation of $\Delta b_i$ | 0.66 |

In order to determine if there were significant differences between the $b_i^+$ and $b_i^-$ estimates for individual securities, a t value was calculated as follows;

$$t_{\Delta b_i} = \frac{\Delta b_i}{\sqrt{\left(\sigma^2_{b_i^+}/n\right) + \left(\sigma^2_{b_i^-}/n\right)}}$$

where $\Delta b_i = b_i^+ - b_i^-$

and $\sigma_{bi}^+$ and $\sigma_{bi}^-$ are the standard error of the estimates. The test of the mean difference showed that 290 of the 322 securities (90% of the sample) exhibited significantly different $b_i^+$ and $b_i^-$'s at the 0.05 level. These results support the assumption that securities may respond differently in up- and down-markets.

Tests of the Two-Beta Model

The two-beta model of equation (7) was tested to determine if the expected negative $\lambda_1$ and positive $\lambda_2$ would be confirmed. Several alternative size portfolios were used in the testing. These portfolios were constructed in the following manner;

(1) The $b_i^+$ and $b_i^-$ values were ranked separately in descending order and a matrix similar to Table 1 was formed,

(2) The rankings, both $b_i^+$ and $b_i^-$ were divided into m equal size groups (m = 3, 4 and 5). This results in m X m portfolios or nine portfolios when m = 3, 16 portfolios when m = 4 and 25 portfolios when m = 5.

This procedure stratifies the sample portfolios based on their responses in both up- and down-markets. For example, the first portfolio includes those securities with the highest $b_i^+$ and $b_i^-$ values (i.e., the lower right-hand corner of Table 1) while the last portfolio includes those securities with the lowest $b_i^+$ and $b_i^-$ values (i.e., upper left-hand corner). While

this procedure stratifies the sample, it does not necessarily assign the same number of securities to each portfolio.

The regression results of equation (7) are shown in Table 2. In addition to the 9, 16 and 25 portfolios produced by the grouping procedure, the 322 single-security portfolio results are also presented.

Table 2 here

As hypothesized, the sign of $\lambda_1$ was negative while the sign of $\lambda_2$ was positive. As can be seen, however, the $\lambda_1$ coefficient is generally statistically weaker than $\lambda_2$. These results suggest that investors do, indeed, expect risk premiums for downside variation of returns and will pay a premium for upside variation.

Table 2 also shows the values of $\lambda_1 + \lambda_2$ which according to equation (8) should equal $E(R_m) - R_f$ if $b_i^+ = b_i^- = 1.0$. However, during this period the average monthly market return was 0.619 percent and the average monthly risk free rate (three-month T-Bills) was 0.407 percent. Therefore, the excess market return was 0.212 percent for the period. Since $b_i^+$ and $b_i^-$ did not equal 1.0, the excess market return can be compared with the values found using equation (7). These values are also provided in Table 2. As can be seen, the values are greater than the excess return partially because the $b_i^+$ and the $b_i^-$ of the sample are greater than the 1.0 of the market portfolio. The two-beta model exhibits strong predictive ability as evidenced by the reasonably high $R^2$ values for the tests involving multi-security portfolios. These values are shown in Table 2.

TABLE 2

RESULTS OF THE TESTS OF THE TWO-BETA MODEL

$$E(R_p) = \lambda_0 + \lambda_{1}b_p^{+} + \lambda_{2}b_p^{-}$$

($E(R_m)$ = 0.619 percent was used as the cutoff level
for up- and down-markets.)

| Test Number | Number of Portfolios | $\lambda_0$ | $\lambda_1$ | $\lambda_2$ | $R^2$ | $\lambda_{1}b_p^{+} + \lambda_{2}b_p^{-}$ |
|---|---|---|---|---|---|---|
| 1 | 9 | 1.223<br>(0.253)*** | -0.304<br>(0.164)* | 0.674<br>(0.134)*** | 0.824 | .464 |
| 2 | 16 | 1.243<br>(0.225)*** | -0.307<br>(0.141)** | 0.655<br>(0.117)*** | 0.733 | .437 |
| 3 | 25 | 1.206<br>(0.269)*** | -0.353<br>(0.168)** | 0.746<br>(0.137)*** | 0.608 | .494 |
| 4 | 322 | 1.326<br>(0.192)*** | -0.355<br>(0.122)*** | 0.629<br>(0.098)*** | 0.132 | .348 |

Standard error of estimates are presented in parentheses beneath the coefficients.

* Significant at the 10% level
** Significant at the 5% level
*** Significant at the 1% level

## IV. CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to investigate the hypothesis that securities may respond differently in up- and down-markets. Up- and down-market betas were calculated for each of 322 securities. A mean difference test indicated most of the securities exhibited betas which were significantly different in up- and down-markets.

A two-beta model incorporating the up- and down-market responses of a security was developed. This model allowed total systematic risk to be separated into variation due to upside responses which may be viewed as favorable, and variation due to downside responses which is viewed as unfavorable. The signs of the regression coefficients were correct and statistically significant, indicating investors received a premium for accepting downside risk. Similarly, a negative premium was associated with the up-market beta.

These results suggest that downside risk as measured by $b_p^-$ may be a more appropriate measure of portfolio risk than the conventional single beta.

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